NUMERICAL SOLUTION OF THE TWO-DIMENSIONAL NONSTATIONARY PROBLEM OF THE MOTION OF A SHELL UNDER THE ACTION OF DETONATION PRODUCTS

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UDC 533.6.01.011

The problem of the motion of an incompressible cylindrical shell with an explosive charge is solved numerically for the propagation of a plane detonation wave from the end of the charge. The strength of the shell is not taken into account. A three-term equation of state [1] is assumed for the detonation products. A comparison is made with the one-dimensional case.

A cylindrical shell with an explosive charge (EC) is considered. A plane detonation wave is excited at the open end. The detonation products (DP) escape into a vacuum. The charge is bounded on the left by a rigid wall. After the impact of the detonation wave (DW) on the wall a reflected shock wave appears, and the subsequent motion of the gas is nonisentropic. For a given equation of state of the DP the controlling parameters of the problem will be

$$\lambda = l/R_0, \quad \mu = m / M$$

where m is the mass of the charge, M is the mass of the shell, l is the length, and R_0 is the initial radius of the charge.

The explosive is pentolite (a 50–50 alloy of trinitrotoluene and pentaerythrityl tetranitrate) with an initial density $\rho_0 = 1.65$ g/cm^3, a heat of explosive transformation Q = 0.0536 Mbar · cm^3/g, and a detonation rate D = 0.7655 cm/μsec.

The parameters at the Chapman–Jouguet point are

$$\begin{aligned} p_{c-J} &= 0.2452 \ \text{Mbar} & D &= 0.7655 \ \text{cm}/\mu\text{sec} \\ \rho_{c-J} &= 2.2100 \ \text{g/cm}^3 & v_{c-J} &= 0.1941 \ \text{cm}/\mu\text{sec} \\ e_{c-J} &= 0.0724 \ \text{Mbar} \cdot \text{cm}^3/\text{g} & c_{c-J} &= 0.5714 \ \text{cm}/\mu\text{sec} \end{aligned}$$

where p is the pressure, ρ is the density, e is the internal energy, v is the axial component of velocity, and c is the velocity of sound.

Dimensionless variables are introduced in such a way that the equations of motion and the equation of state are not changed in form:

$$p' = p/\rho_0 D^2, \ \rho' = \rho/\rho_0, \ v' = v/D, \ u' = u/D, \ c' = c/D, \ e' = e/D^2,$$
$$r' = r/R_0, \ z' = z/R_0, \ t' = tD/R_0$$

where u is the radial component of velocity, r is the radial coordinate, z is the axial coordinate, and t is the time.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 76–79, July–August, 1972. Original article submitted January 5, 1971.

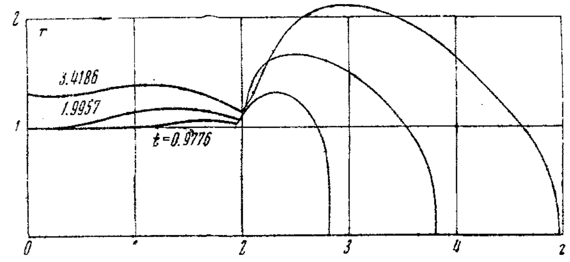

Fig. 1

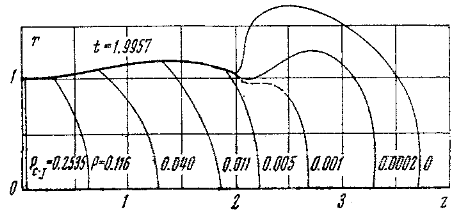

Fig. 2

The equation of state for the detonation products is taken in the form

$$p' = A\rho' e' + B\rho'^4 + C\exp(-k/\rho')$$

The values of the constants are

$$A = 35\cdot 10^{-2},\ B = 1659\cdot 10^{-5},\ C = 2147\cdot 10^{-3},\ k = 3636\cdot 10^{-3}$$

The equation of motion of the shell in dimensional variables is written in the form

$$dM\, d\mathbf{W}/dt = pds\,\mathbf{n}$$

Omitting primes, this expression in component form in dimensionless variables is

$$dU/dt = pR\mu,\ dV/dt = pR\mu\ \mathrm{tg}\,\gamma \tag{1}$$

Here **W** is the velocity of the mass dM, U and V are, respectively, the vertical and horizontal components of the velocity, γ is the angle between the vertical and the surface of the shell, and **n** is a unit vector along the normal to the shell.

The boundary condition on the shell is

$$W_n = w_n \tag{2}$$

where W_n and w_n are the projections of the velocities of the shell and of the detonation products on the normal to the shell.

The boundary conditions on the front of the escaping gas are

$$p = 0,\ \rho = 0$$

Because of the axial symmetry of the problem the radial component of velocity on the axis is zero. Up to the incidence of the detonation wave on the rigid wall the boundary conditions on the wave front have the form

$$p = p_{c-J},\ \rho = \rho_{c-J},\ e = e_{c-J},\ v = v_{c-J},\ u = 0$$

After the appearance of the reflected shock wave the axial component of velocity at the rigid wall is zero.

The initial conditions were specified by using the plane self-similar distribution behind a detonation wave front obtained by G. P. Men'shikov for the applicable equation of state.

The finite-difference approximation to the equations of motion of the detonation products was made by using an explicit two-step scheme of the second order of accuracy. The one-dimensional version of this scheme is described in detail in [2]. The computational scheme is given there is a generalization of one first used by G. S. Roslyakov and L. A. Chudov in 1962 to solve the problem of supersonic flow around a blunt object [3]. The parameters at the rigid wall were calculated by the general scheme using fictitious points: the quantities sought at the fictitious points were determined by an extension of the computational region, taking account of appropriate boundary conditions. At the boundary with a vacuum the pressure and density were assumed zero, and the axial and radial components of the velocity at these points were found by linear extrapolation, using the two closest points occupied by the cloud of detonation products. The DP parameters (ρ, e, v) at the shell were calculated by a unilateral scheme of the first order of accuracy. The pressure p was determined from the equation of state of the DP. The components of the velocity of the shell U and V were found by Eqs. (1) with second order accuracy. The values of v, ρ, and e on the axis were obtained by parabolic extrapolation.

To ensure the stability of the calculation, the time step was found from the condition

$$\Delta\tau = \min\left\{\begin{array}{l}\min K\Delta r/(|u|+c)\\ \min K\Delta z/(|v|+c)\end{array}\right.$$

where Δr is the step in the radial coordinate and Δz is the step in the axial coordinate.

The quantity K (the Courant number) was taken equal to 0.4. The calculations were performed on a BÉSM-6 computer using a 27×25 net.

The shell and the gas cloud are shown in Fig. 1 at various times ($\mu = 1$, $\lambda = 2$). The part of the shell close to the end receives a relatively small displacement because of the rapid fall of pressure in this region due to the intense outflow of DP. The parts of the shell at the rigid wall begin to move at a later time, but they receive larger initial accelerations as a consequence of the increase in pressure due to the reflection of the DW from the wall. The velocity of the gas front emerging into a vacuum increases from 0.7 to 0.92 on the axis. It is interesting to note that the cloud of DP does not propagate to the left of the end; i.e., there is no inflow of DP into the shell. The isobars for t = 1.9957, i.e., at the instant directly preceding the incidence of the detonation wave on the wall, are shown in Fig. 2. The curvature of the isobars is explained by the intense action of the lateral rarefaction wave. For the same value of the axial coordinate the pressure on the shell is significantly lower than the pressure in the central column of the DP.

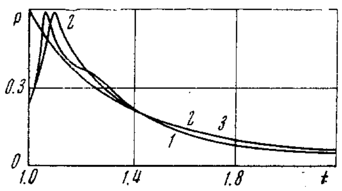

Fig. 3

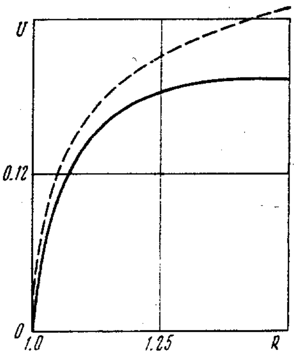

Fig. 4

Figure 3 shows how the pressure varies (curve 1) on the rigid wall after the reflection of the detonation wave in the central column of the DP. Curve 2 shows the numerical solution for plane one-dimensional reflection of the DW for the same equation of state. Curve 3 corresponds to the one-dimensional anayltic solution of K. P. Stanyukovich when the polytrope of the DP has the form $p\rho^{-3}$ = const [4]. Figure 4 shows the radial component of velocity of the shell U as a function of the radius of the shell at the cross section z = 1. The open curve corresponds to the relation $U = f(R)$ for the one-dimensional radial dilation of a shell with cylindrical symmetry when an instantaneous detonation occurs, and the expansion of the detonation products is described by the polytrope $p\rho^{-3}$ = const [4].

The two-dimensional problem of the dilation of a shell was discussed by Wilkins [5] with and without taking account of the strength of the shell. When the strength is not taken into account, Wilkins' results are in qualitative agreement with ours.

The authors thank G. S. Roslyakov and V. M. Paskonov for assistance in the work and for helpful advice.

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